Worked Solutions

Edexcel C4 Paper C

1. (*a*) $\dfrac{5x+7}{(x+1)(x+2)} = \dfrac{2}{x+1} + \dfrac{3}{x+2}$ **(3)**

$y = 2(x+1)^{-1} + 3(x+2)^{-1}$

$\dfrac{dy}{dx} = -2(x+1)^{-2} - 3(x+2)^{-2}$

$\dfrac{d^2y}{dx^2} = 4(x+1)^{-3} + 6(x+2)^{-3}$

when $x = 1$, $\dfrac{d^2y}{dx^2} = \dfrac{4}{2^3} + \dfrac{6}{3^3} = \dfrac{13}{18}$ **(3)**

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2. $V = 36h^2 \Rightarrow \dfrac{dV}{dh} = 72h$

we are given $\dfrac{dV}{dt} = 24$

$\dfrac{dV}{dt} = \dfrac{dV}{dh} \times \dfrac{dh}{dt}$

when $h = 2$, $24 = 72 \times 2 \times \dfrac{dh}{dt}$ $\quad \dfrac{dh}{dt} = \dfrac{24}{144} = \dfrac{1}{6}$ cm s^{-1} **(5)**

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3. (*a*) $(1+2x)^{-\frac{1}{2}} = 1 + \left(-\frac{1}{2}\right)(2x) + \dfrac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}(2x)^2 + \dfrac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{3.2}(2x)^3 + \dots$

$= 1 - x + \dfrac{3}{2}x^2 - \dfrac{5}{2}x^3$ **(4)**

(*b*) valid for $-\dfrac{1}{2} < x < \dfrac{1}{2}$ **(1)**

(*c*) $(1+ax)(1 - x + \frac{3}{2}x^2 - \frac{5}{2}x^3) = 1 - x + \frac{3}{2}x^2 - \frac{5}{2}x^3 + ax - ax^2 + \frac{3a}{2}x^3 + \dots$

$-1 + a = 3, \quad \therefore \quad a = 4$

coef. of $x^3 = -\dfrac{5}{2} + \dfrac{3.4}{2} = \dfrac{7}{2}$ **(4)**

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4. (*a*) when $\theta = \dfrac{\pi}{2}$, $x = \pi + 1$ and $y = 0$

$\dfrac{dy}{d\theta} = -\sin\theta, \qquad \dfrac{dx}{d\theta} = 2 + \cos\theta$

$\dfrac{dy}{dx} = \dfrac{-\sin\theta}{2+\cos\theta}$

when $\theta = \dfrac{\pi}{2}$, gradient of tangent $= \dfrac{-1}{2}$

equation of tangent is $y - 0 = -\dfrac{1}{2}(x - \pi - 1)$

$2y = -x + \pi + 1$

$2y + x = \pi + 1$ **(4)**

(*b*) At stationary points $\dfrac{dy}{dx} = 0, \quad \therefore \quad \sin\theta = 0$

$\theta = 0, \pi, 2\pi$

$\theta = 0, \quad x = 0, \quad y = 1$

$\theta = \pi, \quad x = 2\pi, \quad y = -1$

$\theta = 2\pi, \quad x = 4\pi, \quad y = 1$

There are stationary points at $(0, 1), (2\pi, -1), (4\pi, 1)$ **(4)**

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5. (i)

x	0	1	2	3
$\ln(1+\sin x)$	0	0.61056	0.64674	0.13201

$$I = \frac{1}{2}\left[0 + 0.13201 + 2(0.61056 + 0.64674)\right]$$

$= 1.32$ to 3 sig. fig. **(5)**

(ii) $$\int_0^3 \ln(1+\sin x)^5 \, dx = \int_0^3 5\ln(1+\sin x)dx$$

i.e. $5 \times 1.32 = 6.6$ **(2)**

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6. (*a*) $t = 0, N = 5000$

$5000 = Ae^{\circ}$

$A = 5000$ **(2)**

(*b*) $t = 4, N = 4000 \Rightarrow 4000 = 5000\,e^{-4k}$

$$e^{4k} = \frac{5}{4}$$

$$4k \ln e = \ln\frac{5}{4}$$

$$k = \frac{1}{4}\ln\frac{5}{4} \qquad (0.055786\ldots)$$ **(4)**

(*c*) when $t = 8$, $N = 5000\,e^{-\left(\frac{1}{4}\ln\frac{5}{4}\right)\times 8} = 3200$ **(3)**

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7. (*a*) $(x-3)(x-1)$

let $$\frac{2x}{(x-3)(x-1)} \equiv \frac{A}{x-3} + \frac{B}{x-1}$$

$$= \frac{3}{x-3} - \frac{1}{x-1}$$ (cover up rule) **(3)**

(*b*) $$\frac{1}{y}dy = \int \frac{3}{x-3} - \frac{1}{x-1}\; dx$$

$\ln y = 3\ln(x-3) - \ln(x-1) + c$

$y = \frac{1}{3}, x = 4:\quad \ln\frac{1}{3} = 3\ln 1 - \ln 3 + c$

$c = \ln\frac{1}{3} + \ln 3 = 0$

so $\ln y = \ln\frac{(x-3)^3}{(x-1)}$ $\qquad y = \frac{(x-3)^3}{(x-1)}$ **(6)**

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8. (*a*) direction of l is $\begin{pmatrix} 3 \\ -6 \\ -9 \end{pmatrix}$

equation of l is $r = \begin{pmatrix} 1 \\ 6 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -6 \\ -9 \end{pmatrix}$. **(2)**

(*b*) at point of intersection $1 + 3\lambda = 4 + \mu$

$6 - 6\lambda = 8 + 2\mu$

$1 - 9\lambda = -4 - \mu$

solving we obtain $\lambda = \frac{1}{3}$, $\mu = -2$

point of intersection is $(2, 4, -2)$ **(4)**

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(*c*) $\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ k \\ 5 \end{pmatrix} = \sqrt{6}\ \sqrt{25+k^2+25}\cos 60°$

$5 + 2k - 5 = \sqrt{6}\ \sqrt{50+k^2} \times \frac{1}{2}$

$(4k)^2 = 6(50+k^2)$

$16k^2 = 300 + 6k^2$

$10k^2 = 300$

$k^2 = 30$

$k = \sqrt{30}$ **(3)**

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9. (*a*) $\left(\frac{1}{4}, 0\right)$ **(1)**

(*b*) $\frac{\mathrm{d}y}{\mathrm{d}x} = 2x \cdot \frac{1}{2}(1-4x)^{-\frac{1}{2}}(-4) + 2\sqrt{1-4x}$

$= \frac{-4x}{\sqrt{1-4x}} + 2\sqrt{1-4x} = \frac{-4x+2(1-4x)}{\sqrt{1-4x}} = \frac{2-12x}{\sqrt{1-4x}}$

$\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ at $x = \frac{1}{6}$ and at that point $y = \frac{2}{6}\sqrt{1-\frac{4}{6}} = \frac{1}{3\sqrt{3}}$ **(5)**

(*c*) area $= \int_0^{\frac{1}{4}} 2x\sqrt{1-4x}\ \mathrm{d}x$ $\qquad u = 1-4x, \quad 4x = 1-u$, $\quad \frac{\mathrm{d}u}{\mathrm{d}x} = -4$

$= -\frac{1}{4}\int_1^0 \left(\frac{1-u}{2}\right)u^{\frac{1}{2}}\ \mathrm{d}u$ $\qquad -\frac{1}{4}\mathrm{d}u = \mathrm{d}x$

$= \frac{1}{8}\int_0^1 \left(u^{\frac{1}{2}} - u^{\frac{3}{2}}\right)\mathrm{d}u$ $\qquad x = \frac{1}{4}, \quad u = 0$; $\quad x = 0, \quad u = 1$

$= \frac{1}{8}\left[\frac{2}{3}u^{\frac{3}{2}} - \frac{2}{5}u^{\frac{5}{2}}\right]_0^1$

$= \frac{1}{8}\left[\frac{2}{3} - \frac{2}{5} - 0\right] = \frac{1}{30}$ **(7)**

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